The Math of Gains and Losses
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One of the more compelling aspects of investing is the math of gains and losses. Very simply, a 50% gain does not allow a portfolio to recover from a 50% loss. In fact, a 100% gain is required to restore a 50% loss. Figure 1 demonstrates the needed gain to recover from a variety of different percentage losses.

Figure 1: The Math of Recovery From a Portfolio Loss

Starting account balance	Percentage loss	Ending account balance after loss	Needed percentage gain to restore loss	Ending balance after gain
$1,000	-5%	$950	5.3%	$1,000
1,000	-10%	900	11.1%	1,000
1,000	-15%	850	17.6%	1,000
1,000	-20%	800	25.0%	1,000
1,000	-25%	750	33.3%	1,000
1,000	-30%	700	42.9%	1,000
1,000	-35%	650	53.8%	1,000
1,000	-40%	600	66.7%	1,000
1,000	-45%	550	81.8%	1,000
1,000	-50%	500	100.0%	1,000
1,000	-55%	450	122.2%	1,000
1,000	-60%	400	150.0%	1,000
1,000	-65%	350	185.7%	1,000
1,000	-70%	300	233.3%	1,000
1,000	-75%	250	300.0%	1,000

Source: Craig Israelsen, Ph.D.

The mathematical relationship between losses and gains is a reciprocal one.

Needed % gain to restore loss = [1 / (1 - % Loss)] - 1

For example, the needed gain to restore a 10% loss can be expressed as:

[1 / (1 - .10)] - 1 = .1111 or 11.11%

Thus, the needed gain to restore the 10% loss is 11.11%. As shown in Figure 2, there is a nonlinear relationship between losses and the required subsequent gain needed to recover from the loss. The term "nonlinear" simply means that as the loss gets bigger, the needed gain to restore the loss increases at an increasing rate.

Figure 2: The Steep Road to Recovery
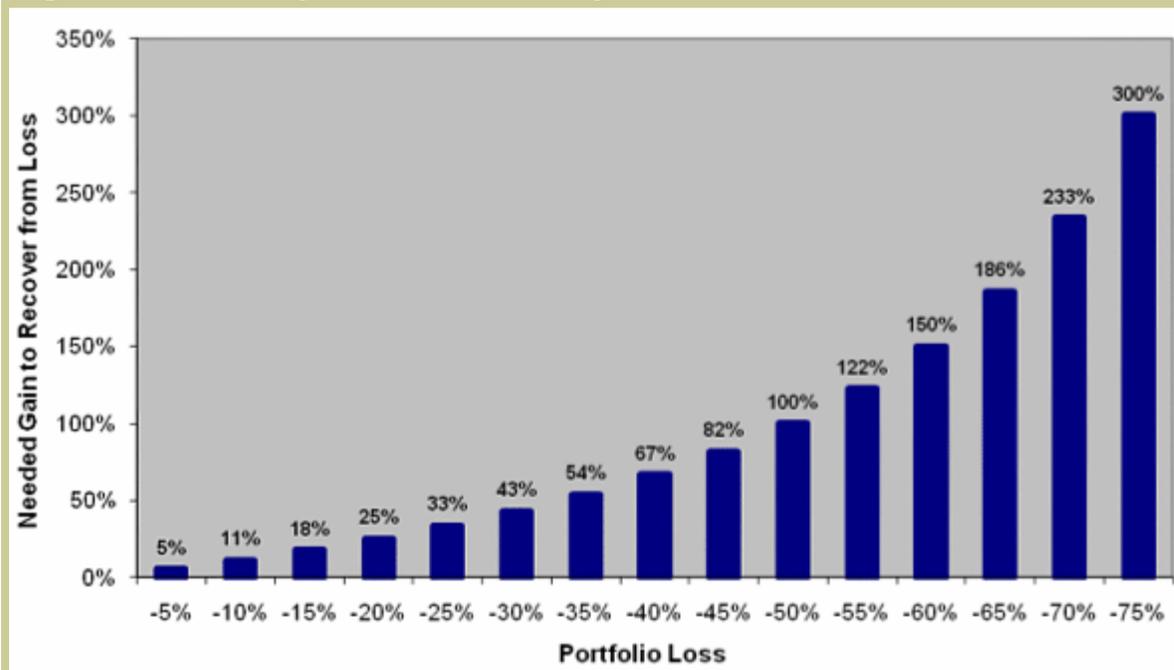
Source: Craig Israelsen, Ph.D.

For example, a portfolio loss of 35% requires a 54% gain to restore the portfolio to whole. So, historically speaking, how long has it taken the S&P 500 to generate a 54% gain?

The long and winding road to recovery

From 1970 through 2009, the S&P 500 Index never had a one-year return in excess of 54%. The largest one-year return was 37.58% in 1995. Therefore, based on historical returns of the S&P 500 Index over the past 40 years, a loss of 35% will require more than one year to recover.

As shown in Figure 3, the S&P 500 Index has a 17.8% chance of gaining 54% (as a cumulative percentage return) within a contiguous two-year period and thus recovering from a 35% loss. The probability of fully recovering from a 35% loss increases to 34.2% over a holding period of three years. There is a nearly 57% chance of recovery over a four-year period, and a 61% chance of recovery over five years. All of these loss recovery estimates are based on the performance of the S&P 500 Index over the past 40 years, from 1970 to 2009. They assume that no money is withdrawn from the account during the recovery period and that no additional money is invested. Taxes and inflation have not been considered in this analysis. And, as always, past performance is not a guarantee of future results.

Figure 3: Probability of Recovery

Portfolio loss	Needed cumulative gain to restore loss	Percentage chance* of recovery from loss within...					
		1 year	2 years	3 years	4 years	5 years	10 years
-10%	11.1%	52.5%	74.4%	81.6%	78.4%	77.8%	93.5%
-20%	25%	25.0%	48.7%	68.4%	67.6%	72.2%	93.5%
-35%	54%	0.0%	17.9%	34.2%	56.8%	61.1%	93.5%
-50%	100%	0.0%	0.0%	7.9%	13.5%	36.1%	80.6%
-65%	186%	0.0%	0.0%	0.0%	2.7%	5.6%	61.3%

*Probabilities calculated from historical returns of the S&P 500 Index over the past 40 years.
Source: Craig Israelsen, Ph.D.

Smaller portfolio losses, such as a 20% loss, are more quickly resolved. The S&P 500 Index generated a single-year gain of 25% or more (25% being the minimum gain needed to restore a portfolio following a loss of 20%) in 10 separate years between 1970 and 2009. Therefore, based on historical return patterns, there is a 25% chance that the S&P 500 could recover from a 20% loss within one year. There is a 72.2% probability of recovery from a loss of 20% within a period of five years and a 93.5% chance of full portfolio recovery (at least in nominal terms) within 10 years.

More serious losses require longer recovery time frames, if recovery is even possible. For instance, a portfolio invested completely in the S&P 500 Index that loses 50% has a 0% chance of recovery within one or two years. In fact, there is only a 7.9% chance of recovery within three years, and a 36.1% chance of recovery within five years.